Coherent Structure Phenomena in Drift Wave–Zonal Flow Turbulence

A. I. Smolyakov,[1] P. H. Diamond,[2] and M. Malkov[2]
[1]Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, S7N 5E2 Canada
[2]Department of Physics, University of California at San Diego, La Jolla, California 92093
(Received 24 May 1999)

Zonal flows are azimuthally symmetric plasma potential perturbations spontaneously generated from small-scale drift-wave fluctuations via the action of Reynolds stresses. We show that, after initial linear growth, zonal flows can undergo further nonlinear evolution leading to the formation of long-lived coherent structures which consist of self-bound wave packets supporting stationary shear layers. Such coherent zonal flow structures constitute dynamical paradigms for intermittency in drift-wave turbulence that manifests itself by the intermittent distribution of regions with a reduced level of anomalous transport.

PACS numbers: 52.35.Mw, 52.35.Ra, 52.55.Dy

Generation of zonal flows by drift waves in plasmas and the analogous Rossby waves in geostrophic fluids is often observed, both in nature and in numerical simulations [1–3] (see also references in [2]). [We define a zonal flow as a poloidal and toroidally symmetric ($q_z = q_\theta = 0$) potential perturbation with a finite radial scale q_r^{-1} significantly larger than the scale of the underlying small-scale turbulence, $q_r \ll k_r$, where $\mathbf{q}$ is the wave vector for the large-scale perturbations; $\mathbf{k}$ is the wave vector of the background small-scale turbulence; r, θ, and z are the radial, poloidal, and toroidal directions of a straight cylindrical tokamak.] Already, earlier numerical simulations of drift-wave turbulence in a tokamak plasma [4–7] indicated the presence of large-scale components of the spectrum which were later identified as zonal flows. Over the past few years, it has been realized [8,9,10] that zonal flows play a major role in controlling the level of anomalous transport due to drift-wave turbulence in magnetic confinement systems. Recent advances in numerical simulations of tokamak plasmas [11] have unambiguously demonstrated that a certain level of $\mathbf{E} \times \mathbf{B}$ flow (in the poloidal direction) triggers a transition to a state with greatly reduced anomalous transport. The suppression of the turbulence by the sheared $\mathbf{E} \times \mathbf{B}$ flow theoretically investigated in Refs. [12–14] has also been confirmed in experiment [15]. These works indicate that zonal flows play a critical role in the dynamics of drift-wave turbulence and its self-regulation [10]. Further development of the theory of zonal flows is imperative for the understanding of the complex dynamics of transport processes in a tokamak. Because of a similarity between equations for drift waves in plasma and Rossby waves in rotating atmospheres, development of the theory of zonal flows is also important in the geophysics context [3].

The underlying mechanism for zonal flow growth in drift-wave turbulence is the inverse cascade process [1,16], i.e., the energy transfer to large scales. As a result, the effects of small-scale fluctuations appear in the large scale not as turbulent damping but as negative viscosity [17–20] that gives rise to the zonal flow instability. In this paper, we study the nonlinear evolution of zonal flows and the underlying small-scale turbulence. Until now, virtually all theoretical analyses of zonal flows have been perturbative and statistical in approach, thus precluding the treatment of coherent structure phenomena. However, as the amplitude of the zonal flows increases, a variety of nonlinear phenomena can occur, such as wave breaking, wave packet trapping, etc. We investigate such phenomena by demonstrating that finite amplitude shear flow perturbations can propagate radially and form kink-type structures describing transitions between two different values of the zonal flow velocity. (These structures are similar to kink solitons of Bloch waves for the magnetization vector in ferromagnetism.) On other hand, the small-scale turbulence is also affected by the large-scale shear flow via a refraction of the ray trajectories for the wave-action density [10,21]. Thus, for a sufficiently large amplitude of the zonal flow perturbations, the wave packets may be trapped, so that stationary Bernstein-Greene-Kruskal (BGK)–type solutions for the wave quanta density can be realized. We quantitatively characterize such self-trapped wave packets and their stability.

We consider zonal flows dynamics within a simple model of drift-wave turbulence described by the equation [22]

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla\right)\frac{e\tilde{\phi}}{T_e} + \mathbf{V}_* \cdot \nabla \frac{e\tilde{\phi}}{T_e} - \rho_s^2\left(\frac{\partial}{\partial t} + \mathbf{V}_0 \cdot \nabla + \tilde{\mathbf{V}}_E \cdot \nabla\right)\nabla_\perp^2 \frac{e\phi}{T_e} = 0. \qquad (1)$$

Here, $\rho_s^2 = T_e/m_i\omega_{ci}^2$ is the ion-sound Larmor radius, $\mathbf{V}_* = \hat{\theta} cT_e/eB_0L_n$ is the electron diamagnetic drift velocity, and L_n is the characteristic length scale of plasma inhomogeneity. The electrostatic potential ϕ is represented as a sum of fluctuating $\tilde{\phi}$ and mean $\bar{\phi}$ quantities, $\mathbf{V}_0 = c\mathbf{b} \times \nabla\bar{\phi}/B_0$, $\tilde{\mathbf{V}}_E = c\mathbf{b} \times \nabla\tilde{\phi}/B_0$. The large-scale plasma flow $\mathbf{V}_0$ varies on a longer time scale compared to the small-scale fluctuations, so that we may employ a multiple scale expansion, thus assuming that there is a sufficient spectral gap separating large-scale ($\mathbf{X}$, T) and small-scale

$(\mathbf{x},\ t)$ motions. In some situations, the space scale separation may not be so clearly pronounced [5,6]. However, the small-scale components of the zonal flows with $q_r \sim k_r$ are in general less important, as shown in Refs. [10,14].

The model given by Eq. (1) is similar to the Hasegawa-Mima model except the different treatment of the mean component $\bar{\phi}(\mathbf{X}, T)$ which does not enter the first term in (1) because plasma density does not follow the Boltzmann distribution for the zonal flow modes with $q_z = q_\theta = 0$. Note that it is the total electrostatic potential ϕ that enters the last term in (1).

Averaging (1) over the magnetic surface and over fast small scales, we obtain the evolution equation for the mean flow:

$$\frac{\partial}{\partial T} \nabla_\perp^2 \bar{\phi} = -\frac{c}{B_0} \overline{\mathbf{b} \cdot \nabla\tilde{\phi} \times \nabla\nabla_\perp^2 \tilde{\phi}} - \gamma_d \nabla_\perp^2 \bar{\phi}$$

$$= \frac{c}{B_0} \nabla_r^2 (\overline{\nabla_r \phi \nabla_\theta \phi}) - \gamma_d \nabla_\perp^2 \bar{\phi}\,, \tag{2}$$

where the last term describes the flow damping due to plasma collisions [23]. We assume that the mean flow is one dimensional $\bar{\phi}(r, T)$, so that $\nabla_\theta \to 0$, while the small-scale fluctuations are two dimensional, $\tilde{\phi} = \tilde{\phi}(r, \theta)$.

Calculation of the mean quantity $\overline{\nabla_r \phi \nabla_\theta \phi}$ is most conveniently done by employing the kinetic equation for the wave action [10,19,21,24]:

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_k}{\partial \mathbf{k}} \cdot \frac{\partial N_k}{\partial \mathbf{x}} - \frac{\partial \omega_k}{\partial \mathbf{x}} \cdot \frac{\partial N_k}{\partial \mathbf{k}} = 0\,, \tag{3}$$

where $N_k = \mathcal{E}/\omega_k^l \propto (1 + k_\perp^2 \rho^2)^2 |\phi_k|^2$ is the adiabatic action invariant [22], $\mathcal{E}$ is the wave energy, and $\omega_k = k_\theta V_0 + \omega_k^l = k_\theta V_0 + k_\theta V_*/(1 + k_\perp^2 \rho^2)$ is the wave frequency. Note that it is the local frequency ω_k^l (without the Doppler shift) that enters the action invariant. We take $k_\theta = \text{const}$ for azimuthally symmetric flows.

The instability of the zonal flow can be obtained by linearizing Eqs. (2) and (3) for small perturbations $(\tilde{N}_k, \bar{\phi}) \sim \exp(-i\Omega T + iqr)$; $q \equiv q_r = -i\partial/\partial r$ is the radial wave vector of the large-scale perturbation. The instability is related to the in-phase (resonant) part of $\tilde{N}_k$ which is calculated from (3):

$$\tilde{N}_k^r = \frac{\partial}{\partial r} (k_\theta V_0) \frac{\partial N_k}{\partial k_r} R(\Omega, q, \Delta\omega_k)\,. \tag{4}$$

Here, $R(\Omega, q, \Delta\omega_k) = i/(\Omega - qV_g + i\Delta\omega_k)$ is the response function, and $\Delta\omega_k$ is the nonlinear broadening increment, $V_g = \partial\omega/\partial k_r$. In the weakly nonlinear regime $R(\Omega, q, \Delta\omega_k) \to \pi\delta(\Omega - qV_g)$. For the wide spectrum of fluctuations, one obtains $R(\Omega, q, \Delta\omega_k) \to 1/\Delta\omega_k$.

Using (4) in (2), one finds the growth rate $\gamma = q^2 D_{rr}$ of the zonal flow instability [11]:

$$D_{rr} = -\left(\frac{c}{B_0}\right)^2 \int \frac{R(\Omega, q, \Delta\omega_k) k_y^2 k_r}{(1 + k_\perp^2 \rho_s^2)^2} \frac{\partial N_k}{\partial k_r} d^2k\,. \tag{5}$$

Note that the condition for growth is $k_r \partial N_k/\partial k_r < 0$, which is typically satisfied in drift-wave turbulence. Equation (5) describes an initial (linear) stage of zonal flow growth due to the resonant interaction. For typical tokamak parameters $\Omega \ll \mathbf{q} \cdot \partial\omega_k/\partial\mathbf{k}$, so that the nonresonant response $\tilde{N}_k^{(1)}$ from (3) is

$$\tilde{N}_k^{(1)} = \left(\frac{\partial \omega_k}{\partial k_r}\right)^{-1} k_\theta V_0 \frac{\partial}{\partial k_r} N_0\,. \tag{6}$$

As the amplitude of the zonal flow increases, nonlinear effects become important. The nonlinear response can be determined to the next order:

$$\frac{\partial \tilde{N}_k^{(2)}}{\partial r} = \frac{1}{2} (k_\theta V_0)^2 \left(\frac{\partial \omega_k}{\partial k_r}\right)^{-1} \frac{\partial}{\partial k_r} \left[\left(\frac{\partial \omega_k}{\partial k_r}\right)^{-1} \frac{\partial N_0}{\partial k_r}\right]. \tag{7}$$

Substituting $\tilde{N}_k = \tilde{N}_k^r + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$ into (2), we obtain the nonlinear equation for the evolution of the zonal flow:

$$\frac{\partial}{\partial t} \frac{\partial}{\partial r} V_0 - u \frac{\partial^2}{\partial r^2} V_0 - b \frac{\partial^2}{\partial r^2} V_0^2 = -D_{rr} \frac{\partial^3}{\partial r^3} V_0\,. \tag{8}$$

The u parameter has a meaning of the radial propagation velocity and is defined by

$$u = \left(\frac{c}{B_0}\right)^2 \int \frac{(V_g)^{-1} k_\theta^2 k_r}{(1 + k_\perp^2 \rho_s^2)^2} \frac{\partial N_0}{\partial k_r} d^2k\,. \tag{9}$$

The nonlinear term b is

$$b = \frac{1}{2} \left(\frac{c}{B_0}\right)^2 \int \frac{(V_g)^{-1} k_\theta^3 k_r}{(1 + k_\perp^2 \rho_s^2)^2} \frac{\partial}{\partial k_r} \left[(V_g)^{-1} \frac{\partial N_0}{\partial k_r}\right] d^2k\,. \tag{10}$$

Cooperative effects of wave motion, wave steepening, and instability create a possibility of stationary or moving "switching" wave (kink soliton) which is a transition layer between two different values of the mean flow. The simplest solution of this type can be obtained from (8) by neglecting the time dependent term on the left. Then (8) can be integrated twice to obtain

$$uV_0 + bV_0^2 = D_{rr} \frac{\partial}{\partial r} V_0 + C\,. \tag{11}$$

The integration constant C is determined from the boundary conditions $V_0 \to V_1$, $V_0' = 0$, for $r \to -\infty$, and $V_0 \to V_2$, $V_0' = 0$, for $r \to \infty$. From (11) we find

$$V = \frac{1}{2} [V_1 + V_2 + (V_1 - V_2) \tanh(-r/\delta)]\,, \tag{12}$$

where $\delta = -2D_{rr}/b(V_1 - V_2)$. The values of V_1 and V_2 are related to the coefficients u and b in an obvious way. One can simply generalize this result for a moving structure in the form $V(r - u_0 t)$, leading to a one-parameter family of solutions.

It follows from (8) that, in general, zonal flows are not purely stationary but radially moving structures. Their radial velocity given by Eq. (9) is determined by the value of the radial group velocity as well as by the spectral density of background fluctuations. A simple estimate of u from (9) shows that u is of the order of the drift velocity, $u \simeq |k_\theta/k_r| |e\phi_k/T_e|^2 v_{Te}^2/V_* \simeq v_{Te}\rho_s/L_n = V_*$. For the unstable case, $k_r \partial N_0/\partial k_r < 0$, the radial velocity is opposite to the direction of the group velocity V_{gr}, and inversely proportional to it in amplitude. [Note that our equation is derived for the slow evolving zonal flows with a characteristic frequency Ω which satisfies the inequality $\Omega \ll \mathbf{q} \cdot \mathbf{V}_g$, so the limit $\mathbf{V}_g \to 0$ is not encompassed by expression (9).] This simple analysis demonstrates the self-organization properties of the drift wave–zonal flow system, which lead to the formation of propagating shear layer "domain walls" between regions of constant flow velocity.

The development of the negative diffusion instability and nonlinear wave breaking sometimes leads to finite-time singularities, as reported in [25]. The development of such singularities may be prevented by nonlinearities [similar to that in Eq. (8)] and by higher order diffusive terms which limit the region of unstable q. Higher order diffusive terms (e.g., fourth order hyperviscosity) are obtained from higher orders of the two-scale expansion [20] and from higher order moments responsible for wave-particle (Landau) interactions effects [7]. The nonlinear Eq. (8) with the fourth order derivative term becomes a Kuramoto-Sivashinskii-type equation, which is a typical model for pattern formation in unstable media [26].

Another class of stationary solutions for the system of Eqs. (2) and (3) can be obtained by exploiting its analogy with the collisionless Vlasov equation and the self-gravitating systems [27]. These strongly nonlinear solutions have properties akin to BGK modes of the Vlasov equation and occur due to the reflection and trapping of wave packets by zonal flows [22,28]. As a result, the distribution function of the wave-action density is modified toward a marginal equilibrium state for the zonal flow, i.e., the state with no net growth. As with BGK solutions of the Vlasov-Poisson system, there is an infinity of such solutions. To this end, we adopt the methodology of Dupree [28,29] to simply characterize stable, localized equilibria which are "Jeans marginal" states [27], i.e., states which marginally stable to zonal flow instability. The central idea of such equilibria is an existence of a unique, self-supported scale, namely, the Jeans length [27]. By using a box approximation to the packet's quanta density distribution (i.e., action density), the packet intensity, size, dispersion, and speed may be self-consistently related by an analytically derived marginal stability criterion.

The box approximation for the wave-action density allows us to give simple quantitative relations. Within this approach, the action density is $N_k = N_0 + \tilde{N}(k_\theta)W(k - k_{r,0}, \Delta k_r)$, where $W(k - k_{r,0}, \Delta k_r)$ is a "box" function equal to unity for $|k_r - k_{r,0}| < \Delta k_r/2$ and equal to zero otherwise. Then, the gradient of the action density is

$$\frac{\partial N_k}{\partial k_r} = \frac{\partial N_0}{\partial k_r} + \tilde{N}(k_\theta)\{\delta[k_r - (k_{r,0} - \Delta k_r/2)] - \delta[k_r - (k_{r,0} + \Delta k_r/2)]\}, \tag{13}$$

where $k_{r,0}$ is the wave packet or caviton location, and Δk_r is the bump or hole width. The dispersion Eq. (5) for the instability of zonal flow perturbations has the form

$$\Omega = -q^2 c_s^2 \int dk_\theta \int dk_r\, k_\theta^2 k_r \frac{\partial N_k}{\partial k_r} \frac{i}{\Omega - qV_g + i\Delta\omega_k}. \tag{14}$$

In the marginal state we set $\mathrm{Re}\Omega \to 0$ and include the effect of the damping of the zonal flow γ_d due to ion-ion collisions [9,10], $\mathrm{Im}\Omega = -\gamma_d$, so that the dispersion equation takes the form

$$\gamma_d = -q^2 c_s^2 \int dk_\theta \int dk_r\, k_\theta^2 k_r \frac{\partial N_k}{\partial k_r} \frac{1}{-qV_g + i\Delta\omega_k}. \tag{15}$$

Introducing the response function

$$\epsilon(q) \equiv \gamma_d + q^2 c_s^2 \int dk_\theta \int dk_r\, k_\theta^2 k_r \times \frac{\partial N_0}{\partial k_r} \frac{\Delta\omega_k}{q^2 V_g^2 + \Delta\omega_k^2}, \tag{16}$$

which describes the linear response of zonal flow to the self-bound packet, and taking $\tilde{N}(k_\theta) = \delta(k_\theta - k_{\theta,0})$ for simplicity, we obtain, from (15),

$$\epsilon(q) = -q^2 c_s^2 k_{\theta,0}^2 k_{r,0} \tilde{N}(k_\theta) \times \left[\frac{1}{-qV_g(k_{r,0} - \Delta k_r/2) + i\Delta\omega_k} - \frac{1}{-qV_g(k_{r,0} + \Delta k_r/2) + i\Delta\omega_k}\right]. \tag{17}$$

We approximated $k_r \simeq k_{r,0}$ except in the argument of the group velocity $V_g = V_g(k_r \pm \Delta k_r/2)$. After some simple algebra, Eq. (17) becomes

$$\epsilon(q) = q^2 c_s^2 k_{\theta,0}^2 k_{r,0} \tilde{N} \times \left[\frac{q\Delta k_r V_g'}{(-qV_g + i\Delta\omega_k)^2 - \Delta k_r^2 q^2 V_g'^2/4}\right]. \tag{18}$$

This equation relates the amplitude of the bump/hole $\tilde{N}$ to the wave vector q for the marginal equilibrium states. For a broad fluctuation spectrum, $|qV_g| \ll |\Delta\omega_k|$, and Eq. (18) reads

$$\Delta\omega_k^2 + \Delta k_r^2 q^2 V_g'^2/4 = -\frac{\tilde{N}}{\epsilon(q)} q^2 c_s^2 k_{\theta,0}^2 k_{r,0} (q\Delta k_r V_g'). \tag{19}$$

The dependence on q here is somewhat analogous to the speed-amplitude relation familiar from soliton theory. Taking into account that $V_g' < 0$ here, the self-trapping condition is $\tilde{N} > 0$ for $\epsilon(q) > 0$, and $\tilde{N} < 0$ for $\epsilon(q) < 0$. For a given sign of $\epsilon(q)$, either $\tilde{N} > 0$ or $\tilde{N} < 0$ is selected. Thus the spectrum of the self-trapped wave packet should exhibit definite skewness, depending on radial scale and the scaling of the background spectrum.

Note that effects of the dispersion (Δk_r) and decorrelation ($\Delta\omega_k$) add quadratically in Eq. (19). This is not surprising, as turbulent motions are a natural source of support against self-binding and collapse in self-attracting media. *It is indeed interesting to note that structured solutions persist in the presence of strong dissipation, such as that due to eddy decorrelation* $\Delta\omega_k$ *and collisional flow damping* γ_d.

We have shown in this paper that the condensation of drift waves onto the long wave-length region, a process which is initially described by the negative viscosity instability for the long wavelength component, further leads to the nucleation of large-scale coherent structures. These structures occur both in the large-scale component [as given by Eq. (11)] and in the small-scale background turbulence [as described by Eq. (19)]. The combined effects of the nonlinearity and radial motion cause formations of sharp moving transition fronts switching between two different values of the mean plasma velocity. Similar structures were also observed in numerical simulations in [4,25]. These coherent structures are consistent with a scenario of intermittent regions of strong shear that may be an underlying cause of the transport barriers in the temporal $L \rightarrow H$ dynamics [30] and may give rise to structured, spatially intermittent behavior in generic drift-wave turbulence. Formation of the sharp transition regions is in general agreement with experimental observations of the zonal flow profiles in a number of geostrophic fluids [2]. The picture of localized propagating fronts removing the "supercritical" perturbation from the growth zone is also similar to the avalanche concept from self-organized criticality [31]. It is interesting to note that the nonlinear equation derived in the present paper for the kink structures in zonal flows is the Burgers equation, recently proposed as a simplest prototype model for the avalanche transport event [31]. The other type of structures in the background small-scale turbulence are due to wave packet trapping and reflection. We have characterized the amplitude and the size [Eq. (19)] of the localized wave packets that are in self-consistent stationary equilibrium with zonal flows. These structures should manifest themselves as bumps or holes in the wave-action density spectrum. We have demonstrated that such structures will persist in the presence of dissipative processes such as ion-ion collisions and drift-wave decorrelation. All told, both examples of structure self-organization in drift wave–zonal flow systems constitute dynamical paradigms for the origin of spatiotemporal intermittency in drift-wave turbulence.

This research was supported by NSERC Canada and U.S. DOE Grant No. FG03-88ER53275.

[1] A. Hasegawa and M. Wakatani, Phys. Rev. Lett. **59**, 1581 (1987).
[2] F.H. Busse, Chaos **4**, 123 (1994).
[3] T. Soomere, Phys. Rev. Lett. **75**, 2440 (1995).
[4] R.D. Sydora, V.K. Decyk, and J.M. Dawson, Plasma Phys. Controlled Fusion **38**, A281 (1996).
[5] G. Hammet, M. Beer, W. Dorland, S.C. Cowley, and S.A. Smith, Plasma Phys. Controlled Fusion **35**, 973 (1993).
[6] A.M. Dimits *et al.,* in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomics Energy Agency, Vienna, 1994), Vol. III, p. 457.
[7] R. Waltz, G. Kerbel, and J. Milovich, Phys. Plasmas **1**, 2229 (1994).
[8] P.H. Diamond and Y.-B. Kim, Phys. Fluids B **3**, 1626 (1991).
[9] M.N. Rosenbluth and F.L. Hinton, Phys. Rev. Lett. **80**, 724 (1998).
[10] P.H. Diamond, M.N. Rosenbluth, F.L. Hinton, M. Malkov, J. Fleischer, and A. Smolyakov, in *Proceedings of the 17th IAEA Fusion Energy Conference* (IAEA-CN-69/TH3/1, 1998).
[11] Z. Lin, T.S. Hahm, W.W. Lee, W.M. Tang, and R.B. White, Science **281**, 1835 (1998).
[12] H. Biglari, P.H. Diamond, and P.W. Terry, Phys. Fluids B **2**, 1 (1990).
[13] T.S. Hahm and K.H. Burrell, Phys. Plasmas **2**, 1648 (1995).
[14] T.S. Hahm *et al.,* Phys. Plasmas **6**, 922 (1999).
[15] T.S. Hahm *et al.,* Plasma Phys. Controlled Fusion **40**, 657 (1998); **38**, 1427 (1996).
[16] R.Z. Sagdeev, V.D. Shapiro, and V.I. Shevchenko, Sov. J. Plasma Phys. **4**, 551 (1978).
[17] A.V. Tur, A.V. Chechkin, and V.V. Yanovsky, Phys. Fluids B **4**, 3513 (1992); A.V. Chechkin, M.I. Kopp, V.V. Yanovsky, and A.V. Tur, JETP **86**, 357 (1998).
[18] E.A. Fedutenko, Phys. Rev. E **53**, 493 (1996); Phys. Lett. A **200**, 134 (1995).
[19] A. Muhm, A.M. Pukhov, K.H. Spatchek, and V.N. Tsytovich, Phys. Fluids B **2**, 336 (1992).
[20] A.V. Gruzinov, P.H. Diamond, and V.B. Lebedev, Plasmas **1**, 3148 (1994).
[21] V.B. Lebedev, P.H. Diamond, V.D. Shapiro, and G.I. Soloviev, Phys. Plasmas **2**, 4420 (1995).
[22] N. Mattor and P.H. Diamond, Phys. Plasmas **1**, 4002 (1994); N. Mattor and P.H. Diamond, Phys. Rev. Lett. **72**, 486 (1994).
[23] M.N. Rosenbluth and F.L. Hinton, Plasma Phys. Controlled Fusion **41**, A653 (1999).
[24] A.I. Dyachenko, S.V. Nazarenko, and V.E. Zakharov, Phys. Lett. A **165**, 330 (1995).
[25] D.Yu. Manin and S.V. Nazarenko, Phys. Fluids **6**, 1158 (1994).
[26] M.C. Cross and P.C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[27] Ya.B. Zel'dovich and I.D. Novikov, in *The Structure and Evolution of the Universe* (University of Chicago, Chicago, 1983), Vol. 2, p. 242.
[28] P.W. Terry, P.H. Diamond, and T.S. Hahm, Phys. Fluids B **2**, 2048 (1990).
[29] T.H. Dupree, Phys. Fluids **25**, 277 (1982); Phys. Fluids B **4**, 3101 (1992).
[30] P.H. Diamond, V.B. Lebedev, D.E. Newman, and B.A. Carreras, Phys. Plasmas **2**, 3685 (1995).
[31] P.H. Diamond and T.S. Hahm, Phys. Plasmas **2**, 3640 (1995).